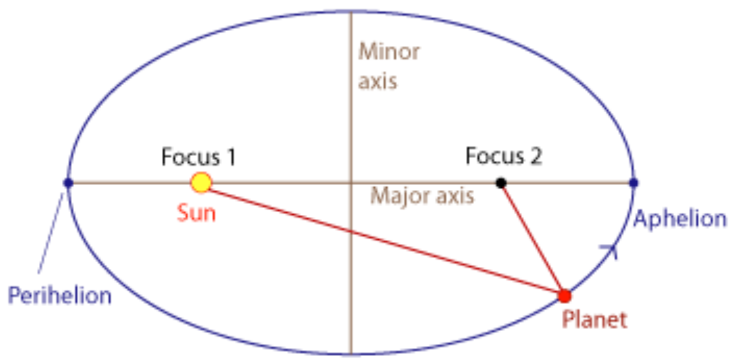
Unit II Fields

Kepler's Laws of Planetary Motion

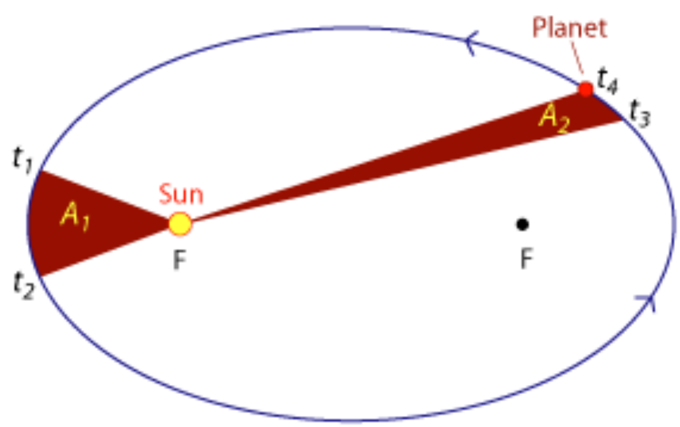
Johannes Kepler (1571 to 1630) made use of the astronomical observations of Tycho Brahe to deduce the following three laws describing the motion of the planets:

Kepler's First Law
The path of each planet around the Sun is an ellipse with the Sun at one focus.



An elliptical orbit of a planet
(greatly exaggerated)

Kepler's Second Law
Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal periods of time.



Kepler's Second Law:
Law of Equal Areas
(greatly exaggerated)

Kepler's Third Law
The ratio of the squares of the periods T of any two planets revolving around the Sun is equal to the ratio of the cubes of their mean distances r from the Sun.

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

This expression can also be rearranged to give the following:

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

Where the value of $\frac{T^2}{r^3}$ is a constant (called Kepler's constant) for every satellite orbiting the same object.

Note:

1. Any units can be used for T and r, as long as you are consistent throughout each problem.
2. The Astronomical Unit (AU) is a useful measure of distance inside the solar system. It is equal to the average distance from Earth to the Sun. $\left(1\ AU = 1.5 \times 10^{11}\ m\right)$

Example 1 Where is Mars?
Mars' period was noted by Kepler to be about 687 days, which is 1.88 years. Determine the distance of Mars from the Sun using the Earth as a reference.

Example 2 The moons of Jupiter.

Galileo discovered four moons of Jupiter. Io, which he measured to be 4.2 units from the center of Jupiter, has an orbital period of 1.8 days. He measured the radius of Ganymede's orbit as 10.7 units. Find the period of Ganymede's orbit.

Homework

Kepler's Laws Worksheet

Kepler's Laws Worksheet

1. The radius of Venus' orbit around the sun is $1.1 \times 10^{11}\ m$. How many days are there in Venus' year? (229.37 days)

2. It takes 689.98 days for Mars to revolve around the Sun. What is the radius of Mars' orbit around the Sun? ($2.28 \times 10^{11}\ m$)

3. A planet's mean distance from the Sun is $2 \times 10^{11}\ m$. What is its orbital period? (562.34 days)

4. If a small planet were discovered whose orbital period was twice that of the Earth, how many times farther from the sun would this planet be? (1.59)

5. Using the data from the table of Useful Planetary Data, determine the Kepler constant for any satellite of the Earth. (**Note**: The moon is a satellite of the Earth) ($1.0 \times 10^{-13}\ s^2 / m^3$)

6. Calculate the Kepler constant for each of the planetary systems listed in the chart below.

Planet	**Mass ($m_E = 1$)**	**Satellite**	**Orbital Radius ($\times 10^3$ km)**	**Period (days)**
Earth	1.00	Moon	384.4	27.322
Mars	0.107	Phobos	9.38	0.319
		Deimos	23.46	1.262
Jupiter	318	Thebe	221.9	0.675
		Io	421.6	1.769
		Europa	670.9	3.551
		Elara	11737	259.7
Saturn	95.2	Janis	151.47	0.695
		Mimas	185.54	0.942
		Calypso	294.67	1.888
Uranus	14.6	Miranda	129.4	1.414
		Ariel	191.0	2.520
		Oberon	583.5	13.463
Neptune	1.72	Triton	355.3	5.877
		Nereid	5510	360.21
Pluto	0.002	Charon	19.7	6.387

7. Using a graph, determine whether there is a mathematical relationship between the k values and the masses of the planets in the above chart. Express the relationship as a mathematical equation.

8. A spy satellite is located one Earth radius above the surface of the Earth. What is its period of revolution in hours? ($4.0\ h$)

9. Mars has two moons, Phobos and Deimos (Fear and Panic, the companions of Mars, the god of war). Deimos has a period of 30 h 18 min and a mean distance from the center of Mars of $2.3\times10^{4}\ km$. If the period of Phobos is 7 h 39 min, what mean distance is it from the center of Mars? ($9.2\times10^{6}\ m$)

Answer to #6:

Earth	$1.3\times10^{-14}\ days^2 / km^3$
Mars	$1.2\times10^{-13}\ days^2 / km^3$
Jupiter	$4.2\times10^{-17}\ days^2 / km^3$
Saturn	$1.4\times10^{-16}\ days^2 / km^3$
Uranus	$9.1\times10^{-16}\ days^2 / km^3$
Neptune	$7.7\times10^{-16}\ days^2 / km^3$
Pluto	$5.3\times10^{-12}\ days^2 / km^3$